The supersymmetric standard model from the $\mathbf{Z}_6'$ orientifold?

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Abstract. We construct $\mathcal{N}=1$ supersymmetric fractional branes on the $\mathbf{Z}_6'$ orientifold. Intersecting stacks of such branes are needed to build a supersymmetric standard model. If a,b are the stacks that generate the $SU(3)_c$ and $SU(2)_L$ gauge particles, then, in order to obtain *just* the chiral spectrum of the (supersymmetric) standard model (with non-zero Yukawa couplings to the Higgs mutiplets), it is necessary that the number of intersections $a \cap b$ of the stacks a and b, and the number of intersections $a \cap b'$ of a with the orientifold image b' of b satisfy $(a \cap b, a \cap b') = (2,1)$ or $(1,2)$. It is also necessary that there is no matter in symmetric representations of the gauge group. We have found a number of examples having these properties. Different lattices give different solutions and different physics.

1. INTRODUCTION

Intersecting D-branes provide an attractive, bottom-up route to standard-like model building [1]. In these models one starts with two stacks, a and b with $N_a = 3$ and $N_b = 2$, of D6-branes wrapping the three large spatial dimensions plus 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold) on which the theory is compactified. These generate the gauge group $U(3) \times U(2) \supset SU(3)_c \times SU(2)_L$, and the non-abelian component of the standard model gauge group is immediately assured. Further, (four-dimensional) fermions in bifundamental representations $(\mathbf{N}_a, \overline{\mathbf{N}}_b) = (\mathbf{3}, \overline{\mathbf{2}})$ of the gauge group can arise at the multiple intersections of the two stacks. These are precisely the representations needed for the quark doublets Q_L of the Standard Model. In general, intersecting branes yield a non-supersymmetric spectrum, so that, to avoid the hierarchy problem, the string scale associated with such models must be low, no more than a few TeV. Then, the high energy (Planck) scale associated with gravitation does not emerge naturally. Nevertheless, it seems that these problems can be surmounted [2, 3], and indeed an attractive model having just the spectrum of the standard model has been constructed [4]. It uses D6-branes that wrap 3-cycles of an orientifold T^6/Ω, where Ω is the world-sheet parity operator. The advantage and, indeed, the necessity of using an orientifold stems from the fact that for every stack $a, b, \ldots$ there is an orientifold image $a', b', \ldots$. At intersections of a and b there are chiral fermions in the $(\mathbf{3}, \overline{\mathbf{2}})$ representation of $U(3) \times U(2)$, where the $\mathbf{3}$ has charge $Q_a = +1$ with respect to the $U(1)_a$ in $U(3) = SU(3)_c \times U(1)_a$, and the $\overline{\mathbf{2}}$ has charge $Q_b = -1$ with respect to the $U(1)_b$ in $U(2) = SU(2)_L \times U(1)_b$. However, at intersections of a and b' there are chiral fermions in the $(\mathbf{3}, \mathbf{2})$ representation, where the $\mathbf{2}$ has $U(1)_b$ charge $Q_b = +1$. In general, besides gauge bosons, stacks of D-branes on orientifolds also have chiral matter in the symmetric **S** and antisymmetric **A** representations of the relevant gauge group; both have charge $Q = 2$ with respect to the relevant $U(1)$. For the stack a with $N_a = 3$, $\mathbf{S}_a = \mathbf{6}$ and $\mathbf{A}_a = \overline{\mathbf{3}}$. The former must be excluded on phenomenological grounds, but the latter could be quark-singlet states q_L^c. Similarly, for the stack b with $N_b = 2$, $\mathbf{S}_b = \mathbf{3}$ and $\mathbf{A}_b = \mathbf{1}$. Again, the former must be excluded on phenomenological grounds, but the latter could be lepton-singlet states ℓ_L^c. Suppose that the number of intersections $a \cap b$ of the stack a with b is p, the number of intersections $a \cap b'$ of the stack a with b' is q, and the number of copies of $\mathbf{A}_a = \overline{\mathbf{3}}$ is r. The standard model has 3 quark doublets Q_L, so that to get just the standard-model spectrum we must have $p + q = 3$. The standard model also has a total of 6 quark-singlet states. To get just the standard model spectrum we also require that $6 - r$ of the quark singlets arise from intersections of a with other stacks $c, d, \ldots$ having just a single D6-brane. These belong to the representation $(\mathbf{1}, \overline{\mathbf{3}})$ of $U(1) \times U(3)$ and each has charge $Q_a = -1$. Ramond-Ramond (RR) tadpole cancellation requires that overall Q_a sums to zero. Thus

$$2p + 2q + 2r - (6 - r) = 0 \tag{1}$$

Hence $r = 0$ and we must also exclude the representations $\mathbf{A}_a = \overline{\mathbf{3}}$. Tadpole cancellation also requires that Q_b sums to zero overall. To get just the standard model spectrum we require that there are 3 lepton doublets L arising from intersections of b with other stacks having just a single D6-brane. All have $Q_b = +1$ or $Q_b = -1$. Suppose the number of copies of $\mathbf{A}_b = \mathbf{3}$ is s. Then overall cancellation of Q_b requires that

$$-3p + 3q + 2s \pm 3 = 0 \tag{2}$$

Hence $s = 0 \bmod 3$. In the case that $s = 0$ the solutions are $(p, q) = (1, 2)$ or $(2, 1)$, whereas when $s = \pm 3$ the solutions $(p, q) = (3, 0)$ or $(0, 3)$ are also allowed [5]. (Models with $|s| > 3$ will obviously have non-standard model spectra.) However, states arising as the antisymmetric representation of $U(2)$ do not have the standard-model Yukawa couplings to the Higgs multiplet. Consequently we are only interested in models such as that in [4] with $(a \cap b, a \cap b') = (1, 2)$ or $(2, 1)$.

Despite the attractiveness of that model, there remain serious problems in the absence of supersymmetry. A generic feature of intersecting brane models is that flavour changing neutral currents are generated by four-fermion operators induced by string instantons [6]. The severe experimental limits on these processes require that the string scale is rather high, of order 10^4 TeV. This makes the fine tuning problem very severe, and the viability of such models highly questionable. Further, in non-supersymmetric theories, such as these, the cancellation of RR tadpoles does not ensure Neveu Schwarz-Neveu Schwarz (NSNS) tadpole cancellation. NSNS tadpoles are simply the first derivative of the scalar potential with respect to the scalar fields, specifically the complex structure and Kähler moduli and the dilaton. A non-vanishing derivative of the scalar potential signifies that such scalar fields are not even solutions of the equations of motion. Thus a particular consequence of the non-cancellation is that the complex structure moduli are unstable [7]. One way to stabilise these moduli is for the D6-branes to wrap 3-cycles of an orbifold T^6/P, where P is a point group, rather than a torus T^6. The FCNC problem can be solved and the complex structure moduli stabilised when the theory is supersymmetric. First, a supersymmetric theory is not obliged to have the low string scale that led to problematic FCNCs induced by string instantons. Second, in a supersymmetric theory, RR tadpole cancellation ensures cancellation of the NSNS tadpoles [8, 9]. An orientifold is then constructed by quotienting the orbifold with the world-sheet parity operator Ω. (As explained above, an orientifold is necessary to allow the possibility of obtaining just the spectrum of the supersymmetric standard model.)

Several attempts have been made to construct the MSSM [10, 11, 12, 13] using an orientifold with point group $P = \mathbf{Z}_4$, $\mathbf{Z}_4 \times \mathbf{Z}_2$ or $\mathbf{Z}_6$. The most successful attempt to date is the last of these [13, 14], which uses D6-branes intersecting on a $\mathbf{Z}_6$ orientifold to construct an $\mathcal{N} = 1$ supersymmetric standard-like model using 5 stacks of branes. We shall not discuss this beautiful model in any detail except to note that the intersection numbers for the stacks a, which generates the $SU(3)_c$ group, and b, which generates the $SU(2)_L$, are $(a \cap b, a \cap b') = (0,3)$. In this case it is impossible to obtain lepton singlet states ℓ_L^c as antisymmetric representations of $U(2)$. Further, it was shown, quite generally, that it is impossible to find stacks a and b such that $(a \cap b, a \cap b') = (2,1)$ or $(1,2)$. Thus, as explained above, it is impossible to obtain exactly the spectrum of the (supersymmetric) standard model.

The question then arises as to whether the use of a different orientifold could circumvent this problem. Here we address this question for the $\mathbf{Z}_6'$ orientifold. We do not attempt to construct a standard(-like) MSSM. Instead, we merely see whether there are any stacks a, b that simultaneously satisfy the supersymmetry constraints, the absence of chiral matter in symmetric representations of the gauge groups (see below), which have not too much chiral matter in antisymmetric representations of the gauge groups, and which have $(a \cap b, a \cap b') = (2,1)$ or $(1,2)$. Further details of this work may be found in reference [15].

2. $\mathbf{Z}_6'$ ORIENTIFOLD

We assume that the torus T^6 factorises into three 2-tori $T_1^2 \times T_2^2 \times T_3^2$. The 2-tori T_k^2 $(k = 1,2,3)$ are parametrised by complex coordinates z_k. The action of the generator θ of the point group $\mathbf{Z}_6'$ on the coordinates z_k is given by

$$\theta z_k = e^{2\pi i v_k} z_k \tag{3}$$

where

$$(v_1, v_2, v_3) = \frac{1}{6}(1,2,-3) \tag{4}$$

The point group action must be an automorphism of the lattice, so in $T_{1,2}^2$ we may take an $SU(3)$ lattice. Specifically we define the basis 1-cycles by π_1 and $\pi_2 \equiv e^{i\pi/3}\pi_1$ in T_1^2, and π_3 and $\pi_4 \equiv e^{i\pi/3}\pi_3$ in T_2^2. Thus the complex structure of these tori is given by $U_1 = e^{i\pi/3} = U_2$. The orientation of $\pi_{1,3}$ relative to the real and imaginary axes of $z_{1,2}$ is arbitrary. Since θ acts as a reflection in T_3^2, the lattice, with basis 1-cycles π_5 and π_6, is arbitrary. The point group action on the basis 1-cycles is then

$$\theta \pi_1 = \pi_2 \qquad \text{and} \qquad \theta \pi_2 = \pi_2 - \pi_1 \tag{5}$$

$$\theta\pi_3 = \pi_4 - \pi_3 \qquad \text{and} \qquad \theta\pi_4 = -\pi_3 \tag{6}$$
$$\theta\pi_5 = -\pi_5 \qquad \text{and} \qquad \theta\pi_6 = -\pi_6 \tag{7}$$

We consider "bulk" 3-cycles of T^6 which are linear combinations of the 8 3-cycles $\pi_{i,j,k} \equiv \pi_i \otimes \pi_j \otimes \pi_k$ where $i = 1,2,\ j = 3,4,\ k = 5,6$. The basis of 3-cycles that are *invariant* under the action of θ contains 4 elements $\rho_{1,3,4,6}$, where

$$\begin{aligned}
\rho_1 &= 2(\pi_{1,3,5} + \pi_{2,3,5} + \pi_{1,4,5} - 2\pi_{2,4,5}) && (8)\\
\rho_3 &= 2(-2\pi_{1,3,5} + \pi_{2,3,5} + \pi_{1,4,5} + \pi_{2,4,5}) && (9)
\end{aligned}$$

and similarly for $\rho_{4,6}$ replacing π_5 by π_6 in $\rho_{1,3}$ respectively. Then the general $\mathbf{Z}'_6$-invariant bulk 3-cycle with (co-prime) wrapping numbers (n_k, m_k) of the cycles (π_{2k-1}, π_{2k}) on T^2_k is

$$\Pi_a = A_1\rho_1 + A_3\rho_3 + A_4\rho_4 + A_6\rho_6 \tag{10}$$

where

$$A_1 = (n_1n_2 + n_1m_2 + m_1n_2)n_3 \tag{11}$$
$$A_3 = (m_1m_2 + n_1m_2 + m_1n_2)n_3 \tag{12}$$
$$A_4 = (n_1n_2 + n_1m_2 + m_1n_2)m_3 \tag{13}$$
$$A_6 = (m_1m_2 + n_1m_2 + m_1n_2)m_3 \tag{14}$$

are the "bulk coefficients". If Π_a has wrapping numbers (n^a_k, m^a_k) $(k = 1,2,3)$, and Π_b has wrapping numbers (n^b_k, m^b_k), then, in an obvious notation, the intersection number of the orbifold-invariant 3-cycles is

$$\begin{aligned}
\Pi_a \cap \Pi_b = -4(A^a_1A^b_4 - A^a_4A^b_1) &+ 2(A^a_1A^b_6 - A^a_6A^b_1) + 2(A^a_3A^b_4 - A^a_4A^b_3) - \\
&- 4(A^a_3A^b_6 - A^a_6A^b_3) && (15)
\end{aligned}$$

which is always even.

Besides these (untwisted) 3-cycles, there are also exceptional 3-cycles associated with (some of) the twisted sectors of the orbifold. They arise in twisted sectors in which there is a fixed torus, and consist of a collapsed 2-cycle at a fixed point times a 1-cycle in the invariant plane. We shall only be concerned with those that arise in the θ^3 sector, which has T^2_2 as the invariant plane. There is a $\mathbf{Z}_2$ symmetry acting in T^2_1 and T^2_3 and this has sixteen fixed points $f_{i,j}$ where $i,j = 1,4,5,6$. There are then 32 independent exceptional cycles given by $f_{i,j} \otimes \pi_{3,4}$ from which 8 independent $\mathbf{Z}'_6$-invariant combinations may be formed. They are

$$\epsilon_j \equiv (f_{6,j} - f_{4,j}) \otimes \pi_3 + (f_{4,j} - f_{5,j}) \otimes \pi_4 \tag{16}$$
$$\tilde{\epsilon}_j \equiv (f_{4,j} - f_{5,j}) \otimes \pi_3 + (f_{5,j} - f_{6,j}) \otimes \pi_4 \tag{17}$$

The non-zero intersection numbers for the invariant combinations are given by

$$\epsilon_j \cap \tilde{\epsilon}_k = -2\delta_{jk} \tag{18}$$

TABLE 1. Relation between fixed points and exceptional 3-cycles.

Fixed point $\otimes$ 1-cycle	**Invariant exceptional 3-cycle**
$f_{1,j} \otimes (n_2\pi_3 + m_2\pi_4)$	0
$f_{4,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$m_2\varepsilon_j + (n_2 + m_2)\tilde{\varepsilon}_j$
$f_{5,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$-(n_2 + m_2)\varepsilon_j - n_2\tilde{\varepsilon}_j$
$f_{6,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$n_2\varepsilon_j - m_2\tilde{\varepsilon}_j$

and again these are always even. The relation between the fixed points $f_{i,j}$ and the invariant exceptional cycles is given in Table 1

The embedding $\mathscr{R}$ of the world-sheet parity operator Ω may be chosen to act on the three complex coordinates z_k $(k = 1,2,3)$ as complex conjugation $\mathscr{R}z_k = \bar{z}_k$, and we require that this too is an automorphism of the lattice. This fixes the orientation of the basis 1-cycles in each torus relative to the Re z_k axis. It requires them to be in one of two configurations **A** or **B**. When T^2_1 is in the **A** configuration, π_1 is aligned along the Re z_1 axis, whereas in the **B** configuration it makes an angle of $\pi/6$ below this axis. Similarly for π_3 and T^2_2. In T^2_3 the cycle π_5 is is aligned along the Re z_3 axis in both **A** and **B** configurations. The difference is that in **A** the 1-cycle π_6 aligned along the Im z_3 axis, whereas in **B** it is inclined such that its real part is one half that of π_5. In both cases the imaginary part is arbitrary, and so therefore is the imaginary part of the complex structure U_3 of T^2_3. It is then straightforward to determine the action of $\mathscr{R}$ on the bulk 3-cycles ρ_p $(p = 1,3,4,6)$ and on the exceptional cycles ε_j and $\tilde{\varepsilon}_j$. In particular, requiring that a bulk 3-cycle $\Pi_a = \sum_p A_p\rho_p$ be invariant under the action of $\mathscr{R}$ gives 2 constraints on the bulk coefficients A_p, so that just 2 of the 4 independent bulk 3-cycles are $\mathscr{R}$-invariant. Which 2 depends upon the lattice.

The twist (4) ensures that the closed-string sector is supersymmetric. In order to avoid supersymmetry breaking in the open-string sector, the D6-branes must wrap special Lagrangian cycles. Then the stack Π_a with wrapping numbers (n^a_k, m^a_k) $(k = 1,2,3)$ is supersymmetric if

$$\sum_{k=1}^{3} \phi^a_k = 0 \bmod 2\pi \tag{19}$$

where ϕ^a_k is the angle that the 1-cycle in T^2_k makes with the Re z_k axis. Defining

$$Z^a \equiv \prod_{k=1}^{3} \pi_{2k-1}(n^a_k + m^a_k U_k) \equiv X^a + iY^a \tag{20}$$

where U_k is the complex structure on T^2_k, the condition (19) that Π_a is supersymmetric may be written as

$$X^a > 0,\ Y^a = 0 \tag{21}$$

(A stack with $Y^a = 0$ but $X^a < 0$, so that $\sum_k \phi^a_k = \pi \bmod 2\pi$, corresponds to a (supersymmetric) stack of anti-D-branes.) In our case $T^2_{1,2}$ are $SU(3)$ lattices, and $U_1 = e^{i\pi/3} = U_2$,

TABLE 2. The functions X^a and Y^a. (An overall positive factor is omitted.) A stack a of D6-branes is supersymmetric if $X^a > 0$ and $Y^a = 0$.

Lattice	X^a	Y^a
AAB	$2A_1^a - A_3^a + A_4^a - \frac{1}{2}A_6^a - A_6^a\sqrt{3}\text{Im}\, U_3$	$\sqrt{3}(A_3^a + \frac{1}{2}A_6^a) + (2A_4^a - A_6^a)\text{Im}\, U_3$
ABB and **BAB**	$\sqrt{3}(A_1^a + \frac{1}{2}A_4^a) + (A_4^a - 2A_6^a)\text{Im}\, U_3$	$2A_3^a - A_1^a + A_6^a - \frac{1}{2}A_4^a + A_4^a\sqrt{3}\text{Im}\, U_3$
BBB	$(A_3^a + A_1^a + \frac{1}{2}A_6^a + \frac{1}{2}A_4^a) +$ $+(A_4 - A_6)\sqrt{3}\text{Im}\, U_3$	$\sqrt{3}(A_3^a - A_1^a + \frac{1}{2}A_6^a - \frac{1}{2}A_4^a)$ $+(A_4 + A_6)\text{Im}\, U_3$

as already noted . Thus

$$Z^a = \pi_1\pi_3\pi_5[A_1^a - A_3^a + U_3(A_4^a - A_6^a) + e^{i\pi/3}(A_3^a + A_6^a U_3)] \tag{22}$$

It is then straightforward to evaluate X^a and Y^a for the different lattices. The results for the cases in which T_3^2 is of **B** type are given in Table 2. The (single) requirement that $Y_a = 0$ means that 3 independent combinations of the 4 invariant bulk 3-cycles may be chosen to be supersymmetric. Of these, 2 are the $\mathscr{R}$-invariant combinations. However, unlike in the case of the $\mathbf{Z}_6$ orientifold, in this case there is a third, independent, supersymmetric bulk 3-cycle that is *not* $\mathscr{R}$-invariant.

We noted earlier that the intersection numbers of both the bulk 3-cycles ρ_p $(p = 1,3,4,6)$ and of the exceptional cycles $\varepsilon_j, \tilde{\varepsilon}_j$ $(j = 1,4,5,6)$ are always even. However, in order to get just the (supersymmetric) standard-model spectrum, either $a \cap b$ or $a \cap b'$ must be odd. It is therefore necessary to use fractional branes of the form

$$a = \frac{1}{2}\Pi_a^{\text{bulk}} + \frac{1}{2}\Pi_a^{\text{ex}} \tag{23}$$

where $\Pi_a^{\text{bulk}} = \sum_p A_p\rho_p$ is an invariant bulk 3-cycle, associated with wrapping numbers $(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a)$, as shown in (10). The exceptional branes (in the θ^3 sector) are associated with the fixed points $f_{i,j},\ (i,j = 1,4,5,6)$ in $T_1^2 \otimes T_3^2$, as shown in (16) and (17). If Π_a^{bulk} is a supersymmetric bulk 3-cycle, then the fractional brane a, defined in (23), preserves supersymmetry provided that the exceptional part Π_a^{ex} arises only from fixed points traversed by the bulk 3-cycle. Since the wrapping numbers (n_1^a, m_1^a) on T_1^2 are integers, the 1-cycle on T_1^2 either traverses zero fixed points or two. In the latter case we denote the fixed points by (i_1^a, i_2^a). Similarly for the 1-cycle on T_3^2, where the two fixed points are denoted by (j_1^a, j_2^a). Thus, supersymmetry requires that the exceptional part Π_a^{ex} of a derives from four fixed points, $f_{i_1^a j_1^a}, f_{i_1^a j_2^a}, f_{i_2^a j_1^a}, f_{i_2^a j_2^a}$. The choice of Wilson lines affects the relative signs with which the contributions from the four fixed points are combined to determine Π_a^{ex}. The rule is that

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \to (-1)^{\tau_0^a}\left[f_{i_1^a j_1^a} + (-1)^{\tau_2^a} f_{i_1^a j_2^a} + (-1)^{\tau_1^a} f_{i_2^a j_1^a} + (-1)^{\tau_1^a + \tau_2^a} f_{i_2^a j_2^a}\right] \tag{24}$$

where $\tau_{0,1,2}^a = 0,1$ with $\tau_1^a = 1$ corresponding to a Wilson line in T_1^2 and likewise for τ_2^a in T_3^2. The fixed point f_{i^a, j^a} with 1-cycle $n_2^a\pi_3 + m_2^a\pi_3$ is then associated with the orbifold invariant exceptional cycle as shown in Table 1.

In general, besides the chiral matter in bifundamental representations that occurs at the intersections of brane stacks $a,b,\ldots$, with each other or with their orientifold images $a',b',\ldots$, there is also chiral matter in the symmetric $\mathbf{S}_a$ and antisymmetric representations $\mathbf{A}_a$ of the gauge group $U(N_a)$, and likewise for $U(N_b)$. Orientifolding induces topological defects, O6-planes, which are sources of RR charge. The number of multiplets in the $\mathbf{S}_a$ and $\mathbf{A}_a$ representations is

$$\#(\mathbf{S}_a) = \frac{1}{2}(a\cap a' - a\cap \Pi_{\mathrm{O6}}) \tag{25}$$

$$\#(\mathbf{A}_a) = \frac{1}{2}(a\cap a' + a\cap \Pi_{\mathrm{O6}}) \tag{26}$$

where Π_{O6} is the total O6-brane homology class; it is $\mathscr{R}$-invariant. If $a\cap\Pi_{\mathrm{O6}} = \frac{1}{2}\Pi_a^{\mathrm{bulk}}\cap\Pi_{\mathrm{O6}} \neq 0$, then copies of one or both representations are inevitably present. Since we require supersymmetry, Π_a^{bulk} is necessarily supersymmetric. However, we have observed above that this does not require Π_a^{bulk} to be $\mathscr{R}$-invariant, as Π_{O6} is. Thus, unlike the $\mathbf{Z}_6$ case, in this case $a\cap\Pi_{\mathrm{O6}}$ is generally non-zero. We noted in the Introduction that we must exclude the appearance of the representations $\mathbf{S}_a$ and $\mathbf{S}_b$. Consequently, we impose the constraints

$$a\cap a' = a\cap\Pi_{\mathrm{O6}} \tag{27}$$

$$b\cap b' = b\cap\Pi_{\mathrm{O6}} \tag{28}$$

We also showed that demanding that the $U(1)$ charges Q_a and Q_b sum to zero overall requires that $\#(\mathbf{A}_a) = 0 = \#(\mathbf{A}_b)$, at least if we also demand standard-model Yukawa couplings. However, for the moment we proceed more conservatively. With the constraint (27) the number of multiplets in the antisymmetric representation $\mathbf{A}_a$ is $a\cap\Pi_{\mathrm{O6}}$. For the present we require only that

$$|a\cap\Pi_{\mathrm{O6}}| \leq 3 \tag{29}$$

since otherwise there would again be non-minimal vector-like quark singlet matter. Similarly, using just (28), we only require that

$$|b\cap\Pi_{\mathrm{O6}}| \leq 3 \tag{30}$$

to avoid unwanted vector-like lepton singlets.

3. RESULTS AND CONCLUSIONS

We have shown [15] that, unlike the $\mathbf{Z}_6$ orientifold, at least on some lattices, the $\mathbf{Z}_6'$ orientifold *can* support supersymmetric stacks a and b of D6-branes with intersection numbers satisfying $(a\circ b, a\circ b') = (2,1)$ or $(1,2)$. Stacks having this property are an indispensable ingredient in any intersecting brane model that has *just* the matter content of the (supersymmetric) standard model. By construction, in all of our solutions there is no matter in symmetric representations of the gauge groups on either stack. However, some of the solutions *do* have matter, 2 quark singlets q_L^c or 2 lepton singlets ℓ_L^c, in the antisymmetric representation of gauge group on one of the stacks. This is not possible on the $\mathbf{Z}_6$ orientifold because all supersymmetric D6-branes wrap the same bulk 3-cycle as the O6-planes. In contrast, on the $\mathbf{Z}_6'$ orientifold there exist supersymmetric 3-cycles that do not wrap the O6-planes. Thus, there is more latitude in this case, and the solutions with antisymmetric matter exploit this feature. Unfortunately, however, none of the solutions of this nature that we have found can be enlarged to give just the standard-model spectrum, since the overall cancellation of the relevant $U(1)$ charge cannot be achieved with this matter content. Nevertheless, some of our solutions have no antisymmetric (or symmetric) matter on either stack. We shall attempt in a future work to construct a realistic (supersymmetric) standard model using one of these solutions.

The presence of singlet matter on the branes in some, but not all, of our solutions is an important feature of our results. It is clear that different orbifold point groups produce different physics, as indeed, for the reasons just given, our results also illustrate. The point group must act as an automorphism of the lattice used, but it is less clear that realising a given point group symmetry on different lattices produces different physics. Our results show that different lattices can produce different physics. The observation that the lattice does affect the physics suggests that other lattices are worth investigating in both the $\mathbf{Z}_6$ and $\mathbf{Z}_6'$ orientifolds. In particular, since Z_6 can be realised on a G_2 lattice, as well as on an $SU(3)$ lattice, one or more of all three $SU(3)$ lattices in the $\mathbf{Z}_6$ case, and of the two on $T^2_{1,2}$ in the $\mathbf{Z}_6'$ case, could be replaced by a G_2 lattice. We shall explore this avenue too in future work.

The construction of a realistic model will, of course, entail adding further stacks of D6-branes $c, d, ..$, with just a single brane in each stack, arranging that the matter content is just that of the supersymmetric standard model, the whole set satisfying the condition for RR tadpole cancellation. In a supersymmetric orientifold, RR tadpole cancellation ensures that NSNS tadpoles are also cancelled, but some moduli, (some of) of the complex structure moduli, the Kähler moduli and the dilaton, remain unstabilised. Recent developments have shown how such moduli may be stabilised using RR, NSNS and metric fluxes [16, 17, 18, 19, 20], and indeed Cámara, Font & Ibáñez [21, 22] have shown how models similar to the ones we have been discussing can be uplifted into ones with stabilised Kähler moduli using a "rigid corset". In general, such fluxes contribute to tadpole cancellation conditions and might make them easier to satisfy. In which case, it may be that one or other of our solutions with antisymmetric matter could be used to obtain just the standard-model spectrum. In contrast, the rigid corset can be added to any RR tadpole-free assembly of D6-branes in order to stabilise all moduli. Thus our results represent an important first step to obtaining a supersymmetric standard model from intersecting branes with all moduli stabilised.

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